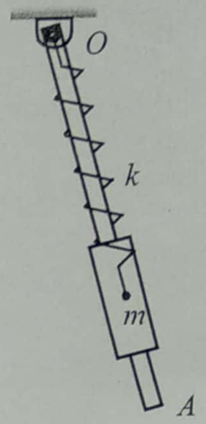
ME 354 A: Vibrations and Controls
19 Feb 2018, Duration: 2 hrs.

All questions are compulsory. Draw neat free body diagrams wherever required. Present your work neatly and clearly state any assumptions made. Springs are massless and non-dissipative.

Q. 1 The vertical resonant amplitude of 4 mm was recorded in a motor with rotating unbalance. At frequencies much higher than at resonance the amplitude of oscillation reached a constant value of 0.8 mm. Determine the damping in the system. (2)

Q. 2 The mass 'm' of a spring-mass system slides without friction on a *horizontally* rotating rod OA with constant angular velocity Ω rad/sec. The unstretched length of the spring is l_0. Now do the following:

(i) Draw a neat free body diagram of the mass. (2)
(ii) Determine the equilibrium position of the mass from O. (2)
(iii) Derive the equation of motion of m about equilibrium position. (5)
(iv) Determine the natural frequency of oscillation of the mass. (2)
(v) Will the contact force between the mass and rod change during oscillations? (2)
(vi) Under what condition the system looses stability? (2)

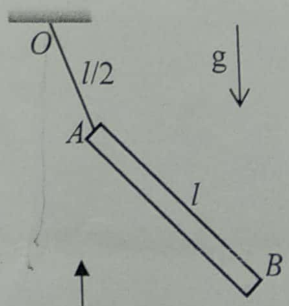


Q. 3 Consider a dynamical system consisting of N particles. Position vector of an i^{th} particle from an inertial frame is $\underline{r}_i = \underline{r}_i(q_1, q_2, \ldots, q_n, t)$, where $i = 1, 2, \ldots, N$, q_k are the generalized independent coordinates and t is time. If mass and acceleration of the i^{th} particle are m_i and $\underline{a}_i$, respectively, then show

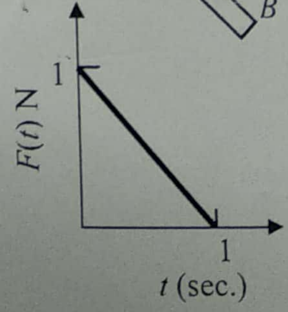
$$\sum_i m_i \underline{a}_i \cdot \delta \underline{r}_i = \sum_k \left(\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} \right) \delta q_k ,$$

where $\delta \underline{r}_i$ is the virtual displacement of the i^{th} particle and T is the total kinetic energy of the dynamical system. (10)

Q. 4 A homogeneous and slender rod AB of mass m is tied to the roof with the help of a massless inextensible string OA. Its motion in confined in a plane. Identify suitable degrees of freedom and find the kinetic and potential energies of the system in terms of known system parameters and the degrees of freedom. For writing the expression of the potential energy use equilibrium position of the system as reference. (8)



Q. 5 A spring-mass system (m = 1 kg and $k = 4\pi^2$ N/m), initially at rest is subjected to a triangular impulse as shown. Determine the expression of displacement of the mass from its equilibrium position using convolution integral. When will be the system's response maximum? (5)



Q. 6 In the following figure a massless rope passes over a circular cylinder and does not slip during motion. Assembly is prepared such that in its equilibrium position spring k_1 is unstretched and mass m_2 has acquired static equilibrium under the influence of gravity. All subsequent motions happen over this state of the system. Now, for the given system parameters do the following:

(i) Identify suitable degrees of freedom and draw neat free body diagrams to derive the equations of motion. (5)

(ii) Derive the equations of motion using Newton – Euler approach. (5)

(iii) For $r = 17.29\pi$ meters, $k_1 = k_2 = 1$ N/m, and $m_1 = m_2 = 1$ kg find the natural frequencies and explain the mode shapes of the system. (5)

(iv) If the cylinder is subjected to a harmonic torque of $T(t) = 10 \sin t$, determine the amplitude of oscillation of m_2. (5)

[Mass moment of inertia of a cylinder of radius r and mass m about its axis $J = 0.5mr^2$.]