$b = \frac{-C(\Delta x_1 + \Delta x_2)}{\Delta x_1 ...}$

$b = -C\Delta x_1 (\Delta x_1 + \Delta x_2)^2 + C(\Delta x_1 + \Delta x_2) = C$

$-C\Delta x_1(\Delta 1 + \Delta x_2) + C = 0.$

max

Computational Fluid Dynamics and Heat Transfer (ME630A) (1.

Department of Mechanical Engineering, Indian Institute of Technology Kanpur

- You may attempt questions in any order. Answer to each question, however, ***must*** start from a fresh page
- Parts of same question ***must*** be answered ***together***, failing which only the first part will be graded
- Multiple answers of same question will incur ***negative*** marks

1. (10 points) We wish to solve the following Equations using both Gauss-Seidel and Jacobi technique. Starting from an initial guess of (0,0), carry out the first three iterations.

$x + 2y = 3$

$2x + y = 3$

$$x + 2y = 3 = 2x + y$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2. (10 points) For the discretization scheme (in a *non-uniform* grid) shown below

Assume

$x = 0, y = 0$

$$\left(\frac{\partial u}{\partial x}\right)_i = au_i + bu_{i+1} + cu_{i+2}$$

a, b, c

$\sum_{j=1}^{n} a_{ij} x_j = b_i$ for $i = 1$

(a) Using Taylor series analysis, find the expressions for the constants a, b and c

$x_p^{(1)} = x_p^{(0)} + \frac{1}{d_p}[b_i -$

(b) Find the order of the truncation error

b & c

2eqn

2 var..

$x_i^{(1)} = (0) + \frac{1}{1}[3-$

3. (20 points) Using Von Neumann stability analysis, find the stability condition for the following numerical scheme (assume uniform grid)

$x_1^{(1)} = 0 + \frac{1}{1}[3-0$

$= 3.$

$x_2^{(1)} = 0 + \frac{1}{1}[3-$

$$\frac{0.5T_i^{n-1} - 2T_i^n + 1.5T_i^{n+1}}{\Delta t} = \alpha \left(\frac{\partial^2 T}{\partial x^2}\right)^n ; \alpha : \text{diffusivity (known constant)}$$

$x_1^{(2)} = \cancel{0} \; 3 + \frac{1}{1}[3-9$

$= -6$

Scanned by CamScanner

$16 - 64 r^2 \sin^2 B/2 + \cancel{\ldots}\ 64 r^2 \sin^2 B/2 . - 12.$

6 $\qquad 16((-2r)-12$

$4 - 64 r \sin^2 B/2 + 64 r^2 \sin^2 B/2.$

4. (30 points) Following Equation governs the x-momentum balance for 2-D, steady, boundary layer flow over a flat plate:

$4(1 - 16 r \sin^2 B_2 + 16 r^2 \sin^2 B_2).$

$\sqrt{(1-16r+16r^2)}$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2};u(y=0)=0,u(y\to\infty)=u(x=0)=u_0$$

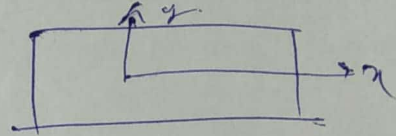
$1 \quad 1 \quad 1$

$0 \quad \Delta x_1 \quad \Delta x_1+\Delta x_2 = 0$

$0 \quad \Delta x_1^2 \quad (\Delta x_1+\Delta x_2)^2 = 0$

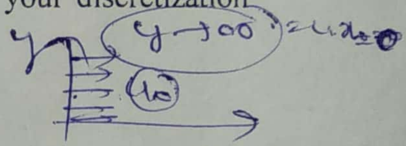
$4(1-2r) \pm$ " $4(1-2r)^2$

Assume μ and ρ to be constant and $v=v(x,y)$ is known, such that the above Equation can now be used for solving u



(a) Comment on the nature of the PDE

(b) Discretize the PDE and the initial/boundary conditions and justify that your discretization conforms to the nature of the PDE



(c) Clearly state (step-by-step) the solution algorithm

5. (30 points) 1-D unsteady heat conduction Equation (in Cartesian coordinate) is discretized (using uniform grid) in the following way:

$$\frac{0.5T_i^{n-1}-2T_i^n+1.5T_i^{n+1}}{\Delta t}=\alpha\left[(1+m)\left(\frac{\partial^2 T}{\partial x^2}\right)^n-m\left(\frac{\partial^2 T}{\partial x^2}\right)^{n-1}\right];\alpha:\text{diffusivity (known constant)}$$

In the above Equation, m is constant that dictates the accuracy of the scheme

(a) Using Taylor series analysis find the modified Equation

PDE → PDE + (error)

(b) Find the order of the truncation error, converting the time-derivatives appearing in the truncation error to spatial derivatives using the following relations:

$\left(\frac{\partial^2 T}{\partial x^2}\right)$

$$\frac{\partial^2 T}{\partial t^2}\approx\alpha^2\frac{\partial^4 T}{\partial x^4};\frac{\partial^3 T}{\partial t\partial x^2}\approx\alpha\frac{\partial^4 T}{\partial x^4};\frac{\partial^3 T}{\partial t^3}\approx\alpha^3\frac{\partial^6 T}{\partial x^6}\text{ etc.}$$

(c) Find the expression of m to make the above scheme 4th-order accurate

$\cancel{u_{xx}} =$

$du_x = u_{xx}\,dx + u_{xy}\,dy$

a b c

Scanned by CamScanner