Feb 23rd, 2018

Computer Aided Engineering Design
Mid-semester Exam

29

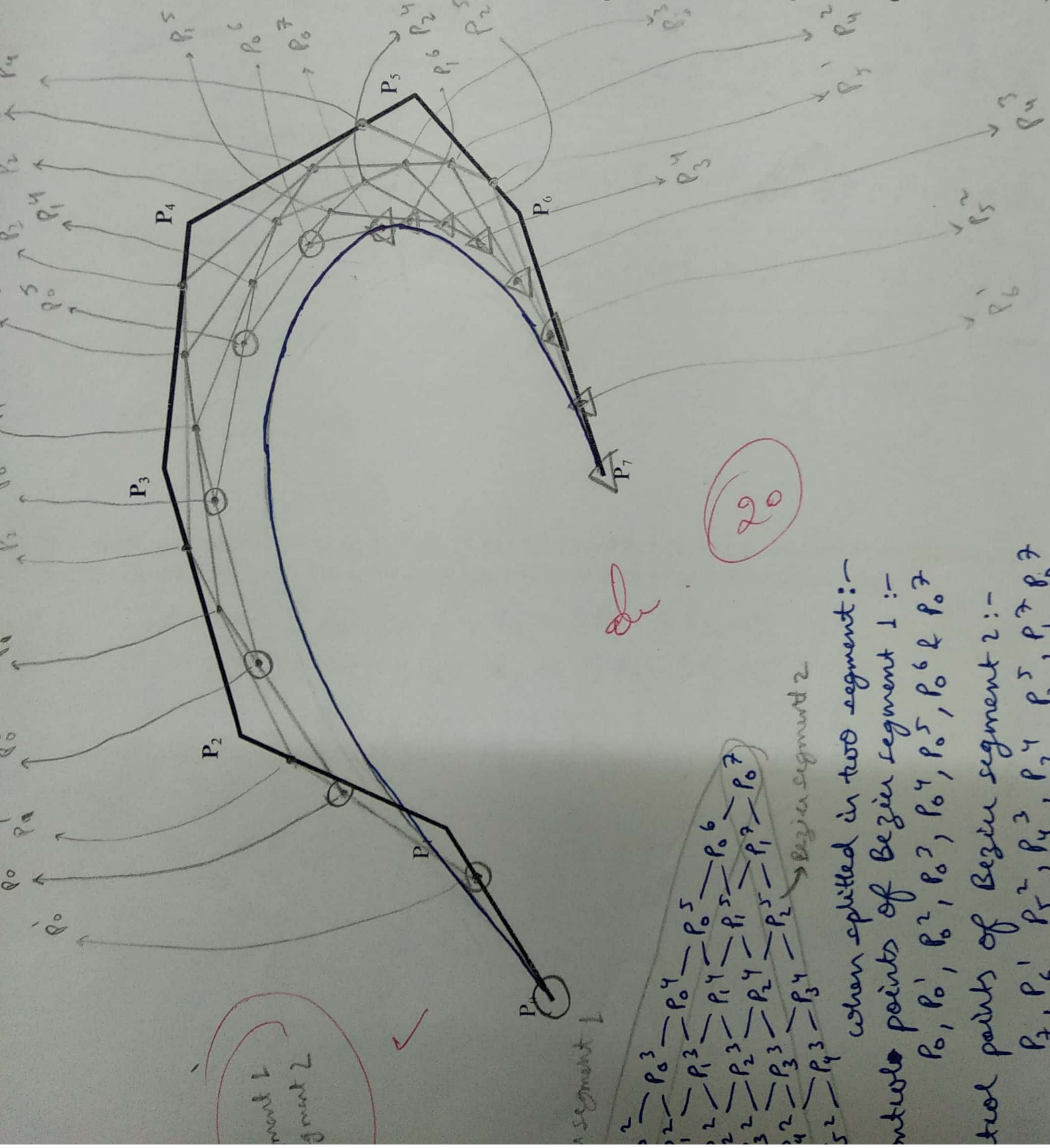
Max Marks: 40

Name: ANUTOSH NIMESH

Roll Number: 150125

Notes: All answers **MUST** be written within the space allocated in the Question paper. Rear portions of the paper should **NOT** be used. The answers may be succinct but must contain ALL important steps. Please be **NEAT**. **BLOCK** important results. For **rough/algebra WORK**, use the institute answer books.

Q1. Given the control polyline, **NEATLY sketch** (orient the paper in the landscape mode) all de Casteljau steps and **label** the intermediate **ALL** de Casteljau points **outside the control polyline**. Sketch the Bézier curve. Use $t = 3/4$. In the space provided below, ARRANGE all de Casteljau points in triangular form. If the curve was to be split into two Bézier segments, identify the respective control polylines in the figure and in the triangular arrangement. (5 + 5 + 5 + 5)



when splitted in two segment :-
control points of Bezier segment 1 :-
$P_0, P_0^1, P_0^2, P_0^3, P_0^4, P_0^5, P_0^6$ & P_0^7
control points of Bezier segment 2 :-
$P_7, P_6^1, P_5^2, P_4^3, P_3^4, P_2^5, P_1^6, P_0^7$

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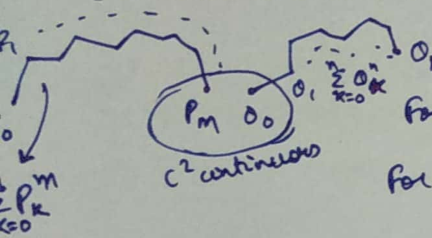
Q2 a. Derive the condition for C^2 continuity in case of composite Ferguson curves. (5)



using:- $\frac{d}{dt} B_i^n(t) = n(B_i^{n-1} + B_{i-1}^{n-1})$; $\frac{d^2}{dt^2} B_i^n(t) = n(n-1)(B_i^{n-2} - 2B_{i-1}^{n-1} + B_{i-2}^{n-2})$

For C^0 continuity :- $P_m = O_0$ — ①

For C^1 continuity :- $m(B_{m-1}^{m-1} P_{m-1} - B_{m-2}^{m-1} P_{m-2}) = n(B_0^{n-1} O_0 - B_1^{n-1} O_1)$ — ②

For C^2 continuity :-

$m(m-1)(B_{m-2}^{m-2} P_{m-2} - 2B_{m-3}^{m-2} P_{m-3} + B_{m-4}^{m-2} P_{m-4}) = n(n-1)(B_0^{n-2} O_0 - 2B_1^{n-2} O_1 + B_2^{n-2} O_2)$ — ③

for cubic curve:- $\boxed{S_{i-1} - 4S_i - S_{i+1} = 3x_{i-1} - 3x_{i+1}}$

$y^?$ (4)

b. Consider data points $\mathbf{P}_0 = (0, 0)$, $\mathbf{P}_1 = (2, 2)$, $\mathbf{P}_2 = (3, 2)$ and $\mathbf{P}_3 = (5, -1)$. Consider the first and last slope vectors as $\mathbf{S}_0 = (1, 1)$, and $\mathbf{S}_3 = (1, -1)$. Using the condition in Q2a, determine the intermediate slopes. (5)

we have $S_0 - 4S_1 + S_2 = 3P_0 - 3P_2$ — ①

$S_1 - 4S_2 + S_3 = 3P_1 - 3P_3$ — ②

$\Rightarrow$ Putting $S_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $S_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $P_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $P_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix}$; $P_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$; $P_3 = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

(3)

we get $4S_2 - S_3 = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

& $S_2 - 4S_3 = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$

$\Rightarrow$ $S_2 = (10/3, 2)$ ✗

& $S_3 = \left(\frac{10}{3}, -2\right)$ ✗

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Q3. A Bézier curve is defined as $\mathbf{r}(t) = \sum_{i=0}^{n} B_i^n(t)\,\mathbf{P}_i$ where $B_i^n(t)$ are the Bernstein polynomials and $\mathbf{P}_i$, the design points. Let $\mathbf{r}_1(u) = \sum_{i=0}^{m} B_i^m(u)\,\mathbf{P}_i$ and $\mathbf{r}_2(v) = \sum_{j=0}^{n} B_j^n(v)\,\mathbf{Q}_j$ be the two Bézier segments. Using the conditions $\frac{d}{du}\mathbf{r}_1(u)\Big|_{u=1} = \lambda \frac{d}{dv}\mathbf{r}_2(v)\Big|_{v=0}$ and $\frac{d^2}{du^2}\mathbf{r}_1(u)\Big|_{u=1} = \lambda \frac{d^2}{dv^2}\mathbf{r}_2(v)\Big|_{v=0} + \beta \frac{d}{dv}\mathbf{r}_2(v)\Big|_{v=0}$ where λ and β are arbitrary scalars, derive the condition for C^2 continuous Bézier curve. Specifically, express **Q**s in terms of **P**s. **(10)**

$$\frac{d}{du} r_1(u)\Big|_{@u=1} = m\left(P_{m-1} - (m-1)P_{m-2}\right) \quad -①$$

$$\frac{d}{dv} r_2(v)\Big|_{@v=0} = n\left(Q_2 - (n-1)Q_1\right) \quad -②$$

$$\text{Eq}^n ① = \lambda\, \text{Eq}^n ② \Rightarrow \boxed{m\left(P_{m-1} - (m-1)P_{m-2}\right) = \lambda n\left(Q_2 - (n-1)Q_1\right)} \quad (I)$$

$$\frac{d^2}{du^2} r_1(u)\Big|_{@u=1} = m(m-1)\left(P_{m-2} - 2(m-2)P_{m-3} + (m-2)(m-3)P_{m-4}\right) \quad -③$$

$$\lambda \frac{d^2}{dv^2} r_2(v)\Big|_{@v=0} = n(n-1)\left(Q_4 - 2(n-2)Q_3 + (n-2)(n-3)Q_2\right) \quad -④$$

(-2)

?

$\text{Eq}^n ③ = \lambda \times \text{Eq}^n ④ + \beta\, \text{Eq}^n ②$:-

$$\boxed{m(m-1)\left(P_{m-2} - 2(m-2)P_{m-3} + (m-2)(m-3)P_{m-4}\right) = \lambda n(n-1)\left(Q_4 - 2(n-2)Q_3 + (n-2)(n-3)Q_2\right) + \beta n\left(Q_2 - (n-1)Q_1\right)} \quad (II)$$

let $r_1(u) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

& $r_2(u) = b_0 + b_1 t + b_2 t^2 + b_3 t^3$

$$(a_1 + 2a_2 + 3a_3) = \lambda b_1 \quad -①$$

$$2a_2 + 3a_3 = \lambda(2b_2) + \beta(b_1) - ②$$

we can solve ① & ② assume $a_0 = b_0$

?